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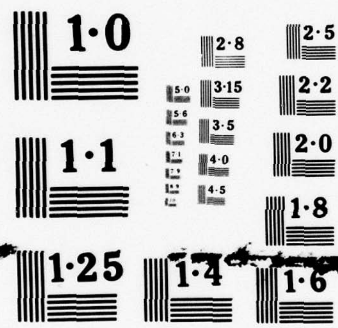

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FOREIGN TECHNOLOGY DIVISION

THEORY OF THIN WING IN A SUPERSONIC FLOW WITH CONSIDERATION OF THE NON-EQUILIBRIUM STATE OF EXCITATION OF OSCILLATING DEGREES OF FREEDOM

by

Ye. P. Aksenov, Yu. N. Grigor'yev

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh^{-1}
cos	cos	ch	cosh	arc ch	cosh^{-1}
tg	tan	th	tanh	arc th	tanh^{-1}
ctg	cot	cth	coth	arc cth	coth^{-1}
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

THEORY OF THIN WING IN A SUPERSONIC FLOW WITH CONSIDERATION OF THE NON-EQUILIBRIUM STATE OF EXCITATION OF OSCILLATING DEGREES OF FREEDOM

Ye. P. Aksenov, Yu. N. Grigor'yev

In rapid gas flows we can observe chemical and thermodynamic non-equilibrium state. It is known that after the action of perturbation, the progressive and rotary components of energy rapidly assume their equilibrium values, and the oscillating component of energy achieves its equilibrium value many times slower; and the relaxation time, as we call this time interval, can prove to be substantial. This permits us to take the following scheme of examining the non-equilibrium state in our problem.

In a shock wave there occurs an initial excitation of the progressive and rotary degrees of freedom under conditions of a frozen state of the oscillating degrees of freedom.

After the shock wave we have non-equilibrium excitation of oscillations under those conditions where there is a place for the equilibrium state between the progressive and rotary degrees of freedom.

It is proposed that total energy E_a of active (i.e. progressive and rotary) degrees of freedom, anywhere in the gas, has an equilibrium value of $\bar{E}_a = C_v^{(a)} T$, and oscillating energy E_v satisfies relationship [2]:

$$\frac{dE_v}{dt} = \omega_0 (\bar{E}_v - E_v)', \tag{1}$$

where $\bar{E}_v$ - equilibrium value of oscillating energy, and $\frac{1}{\omega_0}$ - relaxation time.

The purpose of the present work is the calculation of the non-equilibrium state in excitation of oscillating degrees of freedom in the problem of the steady flow-around of a thin wing of infinite span by a supersonic gas flow.

A system was taken as the initial system of equations, containing the usual equations of gas dynamics with the addition of a relaxation equation (1).

As applied to our problem, this system has the form:

$$\begin{gathered}
v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x}, \\
v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y}, \\
v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + \rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) = 0, \\
v_x \frac{\partial}{\partial x} \left(C_v^{(a)} T + E_v \right) + v_y \frac{\partial}{\partial y} \left(C_v^{(a)} T + E_v \right) = -\frac{p}{\rho} \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right), \\
v_x \frac{\partial E_v}{\partial x} + v_y \frac{\partial E_v}{\partial y} = \omega_0 (\bar{E}_v - E_v), \\
p = R \rho T.
\end{gathered} \tag{2}$$

In these equations, v_x, v_y are the components of macroscopic gas rate, ρ - density, p - pressure, T - absolute temperature, $C_v^{(a)}$ - specific heat with a constant volume, relating to the active degrees of freedom.

The desired hydrodynamic elements should satisfy the boundary conditions on an unknown line of strong disturbance and the flow-around conditions.

Let us propose an approximate analytic solution to this problem which gives prepared formulas for computing the desired functions in any point of the profile. With the solution to the problem in the first approximation, the acquired formulas are similar to the known Akkeret Formulas.

1. Linearized equations and boundary conditions of the problem.

Let us examine a thin slightly-curved profile with sharp edges at small attack angles.

We introduce into the system a coordinate, the beginning of which we place in the front part of the wing; axis x we set along the inflowing flow, and axis y above and perpendicular to axis x.

Since the thin profile barely disturbs the inflowing flow, the desired hydrodynamic elements can be given in the form:

$$\begin{gathered} v_x = V_1 + v_x', \\ v_y = v_y', \\ p = p_1 + p', \\ T = T_1 + T', \\ E_v = \overline{E}_v(T_1) + E_v', \\ \rho = \rho_1 + \rho', \end{gathered} \tag{1.1}$$

where ρ_1, V_1, p_1, and T_1 - values of hydrodynamic elements in the undisturbed flow - constant values.

$v_x', v_y', p', T', E_v', \rho'$ and those produced along the coordinates - smalls of the first order.

Linearizing system (2), i.e. disregarding the terms above of the first order of smallness, we obtain system:

$$V_1 \frac{\partial v_x'}{\partial x} = -\frac{1}{\rho_1}\frac{\partial p'}{\partial x}, \tag{1.2}$$
$$V_1 \frac{\partial v_y'}{\partial x} = -\frac{1}{\rho_1}\frac{\partial p'}{\partial y},$$
$$V_1 \frac{\partial \rho'}{\partial x} + \rho_1\left(\frac{\partial v_x'}{\partial x} + \frac{\partial v_y'}{\partial y}\right) = 0,$$
$$V_1 \frac{\partial}{\partial x}\left(C_v^{(a)} T' + E_v'\right) = -\frac{p_1}{\rho_1}\left(\frac{\partial v_x'}{\partial x} + \frac{\partial v_y'}{\partial y}\right),$$
$$V_1 \frac{\partial E_v'}{\partial x} = \omega_0 \left(C_v^{(\kappa)} T' - E_v'\right),$$
$$p' = R\left(\rho_1 T' + T_1' \rho'\right),$$

where

$$C_v^{(\kappa)} = \left(\frac{\partial \bar{E}_v}{\partial T}\right)_{T_1}.$$

For finding the functions of $v_x', v_y', p', T', E_v', \rho'$ which interest us by integrating system (1.2), we must write down the boundary conditions for these functions which interest us.

Let us begin from conditions on the surface of the nonremovable discontinuity. We agree to provide by subscript 1 the values of hydrodynamic elements up to the nonremovable discontinuity, and by index 2 the values of hydrodynamic values after teh surface of nonremovable discontinuity. Then the momentum theorem, the mass conservation law, and the law of conservation of energy for gas masses passing through the shock wave, are written down in the form of a relationship:

$$-\rho_1 V_{n,1}(V_{x,2} - V_{x,1}) = (p_2 - p_1)\cos(n, x),$$
$$-\rho_1 V_{n,1}(V_{y,2} - V_{y,1}) = (p_2 - p_1)\cos(n, y), \tag{1.3}$$
$$\rho_2 [V_{x,2}\cos(n, x) + V_{y,2}\cos(n, y)] = \rho_1 V_{n,1},$$
$$-\rho_1 V_{n,1}\left(\frac{V_{x,2}^2 + V_{y,2}^2}{2} + E_2 - \frac{V_{x,1}^2 + V_{y,1}^2}{2} - E_1\right) =$$
$$= p_2 [V_{x,2}\cos(n, x) + V_{y,2}\cos(n, y)] - p_1 V_{n,1},$$

where $V_{n,1} = V_{x,1}\,\mathrm{Cos}\,(n,\ x) + V_{y,1}\,\mathrm{Cos}\,(n,\ y)$ projection of speed $\bar{V}_1$ of an undisturbed flow on a normal $\bar{n}$ of the surface of nonremovable discontinuity.

Since the x-axis is directed according to $\bar{V}_1$ of an undisturbed flow, $V_{x,1} = V_1$, $V_{y,1} = 0$. If we designate through φ the angle between the tangent of breakdown and the x-axis, then $\mathrm{Cos}\,(n,\ x) = \mathrm{Sin}\,\varphi$, $\mathrm{Cos}\,(n,\ y) = -\mathrm{Cos}\,\varphi$, and consequently $V_{n,1} = V_1 \sin\varphi$.

Then from (1.3) we find:

$$V_{x,2} - V_1 = -\frac{1}{\rho_1 V_1}(p_2 - p_1),\quad V_{y,2} = \frac{\mathrm{ctg}\,\varphi}{\rho_1 V_1}(p_2 - p_1),$$
$$\rho_2 = \frac{\rho_1^2 V_1^2 \sin^2\varphi}{\rho_1 V_1^2 \sin^2\varphi - (p_2 - p_1)},\quad E_2 - E_1 = \frac{p_2^2 - p_1^2}{2\rho_1^2 V_1^2 \sin^2\varphi}. \tag{1.4}$$

Let us propose $\varphi = \alpha + \Delta\varphi$, где $\sin\alpha = \frac{a_1}{V_1} = \frac{1}{\bar{M}_1}$, (1.5)

a_1 - speed of sound in a disturbed flow.

Let us consider that $\Delta\varphi$ - value of the first order of smallness. Then with accuracy up to smalls of the first order, we have

$$\sin^2\varphi = \sin^2(\alpha + \Delta\varphi) \doteq \frac{1}{\bar{M}_1^2} + \frac{2\sqrt{\bar{M}_1^2 - 1}}{\bar{M}_1^2}\Delta\varphi,$$
$$\mathrm{ctg}\,\varphi = \sqrt{\bar{M}_1^2 - 1} - \bar{M}_1^2\Delta\varphi. \tag{1.6}$$

Taking into account (illegible) and (1.6) from the relationships (1.4), we obtain for the desired functions the following conditions on the line of noremovable discontinuity:

$$v_x' = -\frac{1}{\rho_1 V_1}p', \tag{1.7}$$

$$v_y' = \frac{\sqrt{\bar{M}_1^2 - 1}}{\rho_1 V_1}p'. \tag{1.8}$$

$$\rho' = \frac{\bar{M}_1^2}{V_1^2}p'. \tag{1.9}$$

$$E' = \frac{p_1 \bar{M}_1^2}{\rho_1^2 V_1^2}p'. \tag{1.10}$$

Moreover, considering the frozen state of oscillating degrees of freedom with the passing of gas through the shock wave, we have one condition on the line of discontinuity

$$E_v' = 0. \tag{1.11}$$

Let us direct our attention now to the condition on the profile (to the flow-around condition).
We have

$$v_y = v_x \operatorname{tg} \beta, \tag{1.12}$$

where β is the angle of inclination of the tangent to the profile to the x-axis.

Let us linearize this condition on the profile. Let the equation of the profile be

$$y = \varsigma(x). \tag{1.13}$$

Since the profile is thin and mildly curved, it means that value $\operatorname{tg}\beta = \zeta'(x)$ can be considered small of the first order. Assuming in (1.12) $v_y = v_y'$, $v_x = V_1 + v_x'$ and disregarding the value of the smallness of the second order, we obtain condition on the profile in the form

$$v_y' = V_1 \varsigma'(x). \tag{1.14}$$

Thus, we must integrate the system of equations (1.2) with boundary conditions (1.7)-(1.11) and (1.14).

2. General solution of the linearized system of equations.

Let us reduce the system of equations (1.2) to one equation relative to function p'. With this purpose, we will differentiate the fifth equation of system (1.2) twice along the x-axis. We obtain:

$$\frac{\partial^3 E_v'}{\partial x^3} = \frac{\omega_0}{V_1}\left(C_v^{(\kappa)} \frac{\partial^2 T'}{\partial x^2} - \frac{\partial^2 E_v'}{\partial x^2}\right). \tag{2.1}$$

From teh fourth equation of system (1.2) we find

$$\frac{\partial E_v'}{\partial x} = -\frac{p_1}{\rho_1 V_1}\left(\frac{\partial v_x'}{\partial x} + \frac{\partial v_y'}{\partial y}\right) - C_v^{(a)}\frac{\partial T'}{\partial x}\,. \qquad (2.2)$$

Differentiating (2.2) once with respect to x, we obtain

$$\frac{\partial^2 E_v'}{\partial x^2} = -\frac{p_1}{\rho_1 \rho_1}\left(\frac{\partial^2 V_x'}{\partial x^2} + \frac{\partial^2 V_y'}{\partial x\,\partial y}\right) - C_v^{(a)}\frac{\partial^2 T}{\partial x^2}\,. \qquad (2.3)$$

The third equation of system (1.2) with the use of the latter provides

$$\frac{\partial T'}{\partial x} = \frac{1}{R\rho_1}\frac{\partial p'}{\partial x} + \frac{T_1}{V_1}\left(\frac{\partial v_x'}{\partial x} + \frac{\partial v_y'}{\partial y}\right). \qquad (2.4)$$

Having differentiated (2.4) once with respect to x, we find

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{R\rho_1}\frac{\partial^2 p'}{\partial x^2} + \frac{T_1}{V_1}\left(\frac{\partial^2 v_x'}{\partial x^2} + \frac{\partial^2 v_y'}{\partial x\,\partial y}\right)\,. \qquad (2.5)$$

Let us differentiate the first equation of system (1.2) with respect to x, and the second with respect to y. We find

$$\frac{\partial^2 v_x'}{\partial x^2} = -\frac{1}{\rho_1 V_1}\frac{\partial^2 p'}{\partial x^2}\,, \qquad (2.6)$$

$$\frac{\partial^2 v_y'}{\partial x\,\partial y} = -\frac{1}{\rho_1 V_1}\frac{\partial^2 p'}{\partial y^2}\,. \qquad \mathbf{(2.7)}$$

Placing (2.5), (2.6), and (2.7) in (2.3), we obtain

$$\frac{\partial^2 E_v'}{\partial x^2} = \left[\frac{1}{V_1^2}\left(C_v^{(a)}\frac{T_1}{\rho_1} + \frac{p_1}{\rho_1^2}\right) - \frac{C_v^{(a)}}{R\rho_1}\right]\frac{\partial^2 p'}{\partial x^2} + \\ + \frac{1}{V_1^2}\left(C_v^{(a)}\frac{T_1}{\rho_1} + \frac{p_1}{\rho_1^2}\right)\frac{\partial^2 p'}{\partial y^2}\,. \qquad (2.8)$$

Differentiating (2.8) once with respect to x and placing the obtained equation, along with (2.5) and (2.8) in (2.1), using (2.6) and (2.7), we obtain for function p', after the transformations of coefficients, and equation with partial derivatives of the third order in the form:

$$k_0 a^2 \frac{\partial^3 p'}{\partial x^3} - k_0 \frac{\partial^3 p'}{\partial x\,\partial y^2} + b^2 \frac{\partial^2 p'}{\partial x^2} - \frac{\partial^2 p'}{\partial y^2} = 0, \qquad (2.9)$$

where the following designations are introduced:

(see following page)

$$k_0 = \frac{v_1}{\omega_0} \frac{R + C_v^{(a)}}{R + C_v^{(a)} + C_v^{(n)}} ,$$

$$a^2 = \frac{C_v^{(a)}}{R T_1 (R + C_v^{(a)})} V_1^2 - 1 = \overline{M}_1^2 - 1 , \qquad (*)$$

$$b^2 = \frac{C_v^{(a)} + C_v^{(n)}}{R T_1 (R + C_v^{(a)} + C_v^{(n)})} V_1^2 - 1 = M_1^2 - 1 .$$

Equation (2.9) replaces, in the non-equilibrium case, the normally used wave equation. This tie with the classic equation is quite apparent. When the relaxation time approaches null, from (2.9), as in the particular case, an equation follows for an equilibrium period. In the case where $k_0 \to \infty$ (infinitely larger time of relaxation), we obtain a special case of the classic equation with the Mach number of the "frozen" period. For some non-null finite value of relaxation time the characteristics of equation (2.9) with the form

$$\frac{dy}{dx} = \pm \frac{1}{\sqrt{\overline{M}_1 - 1}}$$

play the same role as the characteristics which are well known for equilibrium periods.

We will search for a solution to the obtained equation (2.9) in the form of an exponential law, which is natural for the relaxation process.

It is easy to see that the functions with the form $e^{\alpha x - \beta y}$ and $e^{\alpha^* x - \beta^* y}$, where α and α^*, β and β^* are complex conjugate numbers, are solutions to equation (2.9) with

$$\beta^2 = \frac{b^2 + k_0 a^2 \alpha}{1 + k_0 \alpha} \alpha^2. \qquad (2.10)$$

Then function

$$p' = Ae^{\alpha x - \beta y} + Be^{\alpha^* x - \beta^* y}, \tag{2.11}$$

where A and B are the random constants, and will also be the solution to equation (2.9).

We propose

$$\alpha = p + qi, \quad \beta = \delta + \lambda i. \tag{2.12}$$

Then (2.11) can be written in the form

$$p' = e^{px - \delta y}\,[C\cos(qx - \lambda y) + D\sin(qx - \lambda y)], \tag{2.13}$$

where C=A+B, D=i(A-B). Substituting (2.12) in (2.10) and separating the real and imaginary parts, we obtain

$$\begin{gathered} \delta^2 - \lambda^2 = F, \\ 2\delta\lambda = G, \end{gathered} \tag{2.14}$$

where

$$\begin{aligned} F &= \frac{[(b^2 + k_0 a^2 p)(p^2 - q^2) - 2k_0 a^2 p q^2](1 + k_0 p)}{(1 + k_0 p)^2 + k_0^2 q^2} + \\ &+ \frac{[k_0 a^2 q (p^2 - q^2) + (b^2 + k_0 a^2 p)\, 2pq]\, k_0 q}{(1 + k_0 p)^2 + k_0^2 q^2} \\ G &= \frac{[k_0 a^2 q (p^2 - q^2) + (b^2 + k_0 a^2 p)\, 2pq](1 + k_0 p)}{(1 + k_0 p)^2 + k_0^2 q^2} - \\ &- \frac{[(b^2 + k_0 a^2 p)(p^2 - q^2) - 2k_0 a^2 p q^2]\, k_0 q}{(1 + k_0 p)^2 + k_0^2 q^2}. \end{aligned} \tag{2.15}$$

From (2.14) we find

$$\delta^2 = \frac{1}{2}\left(F + \sqrt{F^2 + G^2}\right), \quad \lambda^2 = \frac{1}{2}\left(-F + \sqrt{F^2 + G^2}\right). \tag{2.16}$$

Having for function p' the expression (2.13), we find, proceeding by opposite means, the remaining hydrodynamic elements: v_x', v_y', T', ρ', E_y'.

Integrating the first equation of system (1.2), we find

$$\begin{aligned} v_x' = -\frac{1}{\rho_1 V_1} e^{px - \delta y}\,[C\cos(qx - \lambda y) + \\ + D\sin(qx - \lambda y)] + C_1(y), \end{aligned} \tag{2.17}$$

where $C_1(y)$ is the arbitrary function.

From the second equation of system (1.2), placing, instead of ρ', its equation (2.13), we obtain

$$\frac{\partial v_y'}{\partial x} = -\frac{1}{\rho_1 V_1} e^{px-\delta y} [(-\delta C - \lambda D)\cos(qx-\lambda y) + + (\lambda C - \delta D)\sin(qx - \lambda y)]. \tag{2.18}$$

Integrating (2.18) with respect to x, we find

$$v_y' = -\frac{e^{px-\delta y}}{\rho_1 V_1 (p^2+q^2)} \{[C(\lambda p - \delta q) - D(\lambda q + \delta p)]\sin(qx-\lambda y) + + [C(-\delta p - \lambda q) + D(\delta q + \lambda p)]\cos(qx-\lambda y)\} + C_2(y), \tag{2.19}$$

where $C_2(y)$ - arbitrary function.

From the third equation of system (1.2), placing, instread of v_x' v_y', their expressions (2.17) and (2.19), we integrate with respect to x and find

$$\rho' = \frac{e^{px-\delta y}}{V_1^2}[C\cos(qx-\lambda y) + D\sin(qx-\lambda y)] - \frac{\rho_1}{V_1}C_2'(y)x + + \frac{e^{px-\delta y}}{V_1^2(p^2+q^2)^2}[(Cg+Df)\sin(qx-\lambda y) + + (Cf-Dg)\cos(qx-\lambda y)] + C_3(y), \tag{2.20}$$

where

$$g = 2pqF + (q^2-p^2)G,$$
$$f = (p^2-q^2)F + 2pqG, \tag{2.21}$$

and $C_3(y)$ - arbitrary function.

From the last equation in system (1.2) we find

$$T' = \frac{V_1^2 - RT_1}{R\rho_1 V_1^2} e^{px-\delta y}[C\cos(qx-\lambda y) + + D\sin(qx-\lambda y)] + \frac{T_1}{V_1}C_2'(y)\cdot x - - \frac{T_1 e^{px-\delta y}}{\rho_1 V_1^2 (p^2+q^2)^2}[(Cg+Df)\sin(qx-\lambda y) + + (Cf-Dg)\cos(qx-\lambda y)] - \frac{T_1}{\rho_1}C_3(y). \tag{2.22}$$

From the fourth equation of system (1.2), using the fifth, we find

$$E_v' = \left[\frac{c_v^{(a)}(V_1^2 - RT_1)}{\omega_0 R \rho_1 V_1} - \frac{p_1}{\rho_1^2 \omega_0 V_1}\right] e^{px-\delta y}\cdot[(-Cq+Dp)\sin(qx- - \lambda y) + (Cp+Dq)\cos(qx-\lambda y)] - \left(\frac{c_v^{(a)} T_1}{\omega_0 \rho_1 V_1} + + \frac{p_1}{\omega_0 \rho_1^2 V_1}\right)\frac{e^{px-\delta y}}{p^2+q^2}[(Cm+Dn)\sin(qx-\lambda y) +$$

$$+ (Cn - Dm)\cos(qx - \lambda y)] - \frac{c_v^{(\kappa)} T_1 e^{px - by}}{\rho_1 V_1^3 (p^2 + q^2)^2} [(Cg + Df) \times$$

$$\times \sin(qx - \lambda y) + (Cf - Dg)\cos(qx - \lambda y)] + \left(\frac{c_v^{(\kappa)} T_1}{\omega_0} + \right.$$

$$\left. + \frac{c_v^{(\kappa)} T_1}{V_1} x + \frac{p_1}{\rho_1 \omega_e} \right) C_2'(y) + \tag{2.23}$$

$$+ \frac{c_v^{(\kappa)} (V_1^2 - RT_1)}{R\rho_1 V_1^3} e^{px - by} [C\cos(qx - \lambda y) + D\sin(qx - \lambda y)] -$$

$$- \frac{c_v^{(\kappa)} T_1}{\rho_1} C_3(y),$$

where

$$m = qF - pG,$$
$$n = pF + qG. \tag{2.24}$$

Functions $p', v_x', v_y', \rho', T', E_v'$, given in formulas (2.13), (2.17), (2.19), (2.20), and (2.23), satisfy the five equations of system (1.2). Satisfying the fifth equation of system (1.2), we obtain one relationship between the random elements entering into the equation. So, in the general solution to system (1.2) arbitrary functions $C_1(y)$, $C_2(y)$, and $C_3(y)$ enter, as do the random constants C, D, p, and q, which we must find from the boundary conditions.

3. Solution to the problem in linear approximation (search for arbitrary functions).

We find arbitrary functions which entered into the general solution. For this, we have six boundary conditions (1.7)-(1.11), and (1.14).

Conditions (1.7)-(1.11) on the line of nonremovable discontinuity will be written on characteristic $x - ay = 0$, and condition (1.14) will be on the y-axis $y = 0$.

Satisfying condition (1.7), we see that

$$C_1\left(\frac{x}{a}\right) = 0, \tag{3.1}$$

and from condition (1.8) we find

$$C_2\left(\frac{x}{a}\right) = \frac{e^{\frac{x}{a}(pa-\delta)}}{\rho_1 V_1 (p^2+q^2)} \{[C(aq^2+ap^2-\delta p-\lambda q)+D(\delta q-$$
$$-\lambda p)]\cos\frac{x}{a}(aq-\lambda)+[C(\lambda p-\delta q)+D(aq^2+ap^2-\delta p-\tag{3.2}$$
$$-\lambda q)]\sin\frac{x}{a}(aq-\lambda)\}.$$

Satisfying condition (1.9), we find

$$-C_2(y) = \left(\frac{1}{V_1^2}-\frac{1}{a_1^2}\right)e^{y(pa-\delta)}[C\cos y(aq-\lambda)+D\sin y(aq-$$
$$-\lambda)]+\frac{e^{y(pa-\delta)}}{V_1^2(p^2+q^2)^2}[(Cg+Df)\sin(aq-\lambda)y+(Cf-Dg)\operatorname{Cos} aq-\tag{3.3}$$
$$-\lambda)y]-\frac{e^{y(pa-\delta)}}{V_1^2(p^2+q^2)^2}[(CM+DN)\cos(aq-\lambda)y-(CN-$$
$$-DM)\sin(aq-\lambda)y]\,a\,y,$$

where

$$M = a(p^2+q^2)(ap-2\delta)+pF+qG,$$
$$N = a(p^2+q^2)(aq-2\lambda)-qF+pG.\tag{3.4}$$

Condition (1.11) leads to equation

$$e^{px-\delta\frac{x}{a}}\left[\left(AC+BD\right)\cos\left(qx-\lambda\frac{x}{a}\right)+(AD-BC)\sin\left(qx-\right.\right.\tag{3.5}$$
$$\left.\left.-\lambda\frac{x}{a}\right)\right]=0,$$

where

$$A = p(a^2-1)\left(\frac{c_v^{(a)}T_1}{\omega_0\rho_1 V_1}+\frac{p_1}{\rho_1^2\omega_0 V_1}\right)+p\frac{c_v^{(a)}V_1^2}{\omega_0 R\rho_1 V_1}+$$
$$+\frac{c_v^{(a)}}{\rho_1}\frac{a_1^2-RT_1}{Ra_1^2}-2a\delta,\tag{3.6}$$
$$B = q(a^2-1)\left(\frac{c_v^{(a)}T_1}{\omega_0\rho_1 V_1}+\frac{p_1}{\rho_1^2\omega_0 V_1}\right)+q\frac{c_v^{(a)}V_1^2}{\omega_0 R\rho_1 V_1}-2a\lambda,$$

which will be satisfied if we set

$$A=0,\qquad B=0.\tag{3.7}$$

Using condition (1.14), we obtain

$$\frac{1}{a}\rho_1 V_1^2 c'(x) = e^{px}(C\cos qx+D\sin qx).\tag{3.8}$$

Replacing in equation (3.8) qx by qx-λy, we find

$$C\cos(qx-\lambda y)+D\sin(qx-\lambda y)=\frac{\rho_1 V_1^2}{a}e^{-px+p\frac{\lambda}{q}y}\varsigma'\left(x-\frac{\lambda}{q}y\right). \qquad (3.9)$$

then

$$p'=\frac{\rho_1 V_1^2}{a}e^{py\left(\frac{\lambda}{q}-\frac{\delta}{p}\right)}\varsigma'\left(x-\frac{\lambda}{q}y\right). \qquad (3.10)$$

The obtained expression for p' is a generalization on the case of movement of gas under conditions of the absence of thermodynamic equilibrium of the known Akkeret formula for the thin wing and concurs with $\kappa_0\to 0$ and $\kappa_0\to\infty$.

Along the same lines, in the case $\kappa_0\to 0$ from (2.1) it follows that $\beta^2=s^2\alpha^2$ and, consequently,

$$\frac{\lambda}{q}=\frac{\delta}{p}=\sqrt{M_1^2-1}\,. \qquad (3.11)$$

So that multiple $e^{py\left(\frac{\lambda}{q}-\frac{\delta}{p}\right)}$, considering the non-equilibrium state in this case turns to a unit, we obtain the Akkeret formula. In the case $k_0\to\infty$

$$\frac{\lambda}{q}=\frac{\delta}{p}=\sqrt{M_1^{*2}-1}\,,$$

from (3.10) we obtain the Akkeret formula with a Mach number of the "frozen" period.

Knowing p', we can compute the coefficients of lift force and drag resistance and determine in this manner the same contribution to the values of these coefficients, which introduces the non-equilibrium state of excitation of oscillating degrees of freedom.

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